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# Conceptual Design and Workspace Analysis of an Exechon-inspired Parallel Kinematic Machine

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**Abstract** Inspired by the commercial application of the Exechon machine, this paper proposed a novel parallel kinematic machine (PKM) named Exe-Variant. By exchanging the sequence of kinematic pairs in each limb of the Exechon machine, the Exe-Variant PKM claims an arrangement of 2UPR/1SPR topology and consists of two identical UPR limbs and one SPR limb. The inverse kinematics of the 2UPR/1SPR parallel mechanism was firstly analyzed based on which a conceptual design of the Exe-Variant was carried out. Then an algorithm of reachable workspace searching for the Exe-Variant and the Exechon was proposed. Finally, the workspaces of two example systems of the Exechon and the Exe-Variant with approximate dimensions were numerically simulated and compared. The comparison shows that the Exe-Variant possesses a competitive workspace with the Exechon machine, indicating it can be used as a promising reconfigurable module in a hybrid 5-DOF machine tool system.

**Keywords** parallel kinematic machine; Exechon; kinematics; workspace

## 1 Introduction

Machine tool manufacturers are facing global competitiveness characterized by large fluctuation in product demands and increasing product varieties in recent years [1]. To survive in this aggressive environment, the machining systems and/or facilities are required to possess reconfigurable ability [2]. Thanks to the merits of high rigidity and dynamics as well as good flexibility and reconfigurability, the parallel kinematic machines (PKMs) have found their compromising applications in aeronautic and automobile industries. This can be best exemplified by the 5-DOF Tricept and Exechon with serial-parallel architecture. According to the public reporting, various versions of the Tricept have been integrated as a plug-and-play module in machine tools and/or robots [3-7].

As a newly invented 5-DOF PKM, the Exechon machine has been experimentally proven to be competitive in terms of accuracy, reliability, and operation speed [8-12]. Although extensive research and application activities have been carried out on the Exechon [13-15], little attention has been paid to its reconfigurable version.

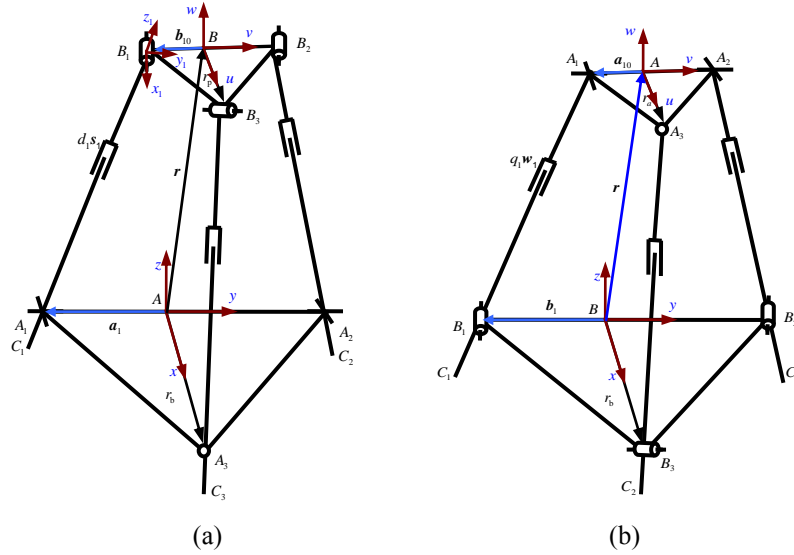
Generally speaking, a reconfigurable PKM module can be realized through the reconfigurability of kinematic chains at the component level [16-19]. The basic ideal of achieving such reconfigurability is to use the combination of a set of readily made limbs that enable the formation of PKM with specified types and DOF. Following this track, a new reconfigurable PKM named Exe-variant is proposed by exchanging the sequence of kinematic pairs in each limb of Exechon machine. To be specific, this Exechon-inspired PKM claims an arrangement of 2UPR/1SPR topology and consists of two identical UPR limbs and one SPR limb.

The reminder of the paper is organized as follows. The inverse kinematics and conceptual design of the Exe-Variant are briefly described in section 2. In section 3, an algorithm of reachable workspace searching is proposed and the workspaces of two example systems of Exechon and Exe-Variant with approximate dimensions are numerically simulated and compared. At last, some conclusions are drawn in section 4.

## 2 Inverse Kinematics and Conceptual Design

### 2.1 Inverse Kinematics

From the topological point of view, the parallel kinematic part of the Exechon machine is a 2RPU/1RPS parallel mechanism while that of the Exe-Variant is a 2UPR/1SPR parallel mechanism. The schematic diagrams of the two parallel mechanisms are depicted in Fig. 1. As the kinematic analysis of the Exechon can be referred to our previous paper [9], the following will only explore the inverse kinematics of the Exe-Variant.



**Fig. 1** Schematic diagrams of (a) Exechon; (b) Exe-Variant

As shown in Fig. 1 (b),  $B_i$  ( $i=1, 2, 3$ ) represents the center of R joint.  $A_i$  ( $i=1, 2$ ) denotes the center of U joint and  $A_3$  is the center of S joint. For the convenience of derivation, some reference frames need to be define at the first place. A global coordinate frame  $B$ -xyz is fixed at the center of  $B_1B_2$ , with  $x$  axis paralleling to  $BB_3$ ,  $y$  axis paralleling to  $B_1B_2$ , while  $z$  axis is determined by the right hand rule. Similarly, a moving coordinate frame  $A$ -uvw is assigned to the moving platform, with the  $u$  axis paralleling to  $AA_3$ ,  $v$  axis paralleling to  $A_1A_2$ .  $r_a$  and  $r_b$  represent the circumcircle radii of the moving platform and the fixed base, respectively.

Assume the transformation matrix of  $A$ - $uvw$  with respect to  $B$ - $xyz$  can be expressed as

$$\mathbf{R} = \begin{bmatrix} c\varphi c\theta & c\varphi s\theta s\psi - s\varphi c\psi & c\varphi s\theta c\psi + s\varphi s\psi \\ s\varphi c\theta & s\varphi s\theta s\psi + c\varphi c\psi & s\varphi s\theta c\psi - c\varphi s\psi \\ -s\theta & s\psi c\theta & c\theta c\psi \end{bmatrix} \quad (1)$$

where “s” and “c” denote “sin” and “cos” functions, respectively;  $\psi$ ,  $\theta$  and  $\varphi$  are Euler angles in terms of precession, nutation and rotation.

The vector of point  $A_i$  measured in  $B$ - $xyz$  can be expressed as

$$\mathbf{a}_i = \mathbf{R}\mathbf{a}_{i0} + \mathbf{r} = \mathbf{b}_i + q_i \mathbf{w}_i \quad (2)$$

where  $\mathbf{a}_{i0}$  denotes the position vector of point  $A_i$  in the frame  $A$ - $uvw$ ;  $\mathbf{r}$  represents the position of point  $A$  in the frame  $B$ - $xyz$ ;  $\mathbf{b}_i$  denotes the vector of point  $B_i$  in the frame  $B$ - $xyz$ ;  $q_i$  and  $\mathbf{w}_i$  represent the distance between  $A_i$  and  $B_i$  and unit vector of the  $i^{\text{th}}$  limb. And there exist

$$\mathbf{a}_{i0} = r_a [\cos\alpha_i \quad \sin\alpha_i \quad 0]^T, \quad \mathbf{b}_i = r_b [\cos\alpha_i \quad \sin\alpha_i \quad 0]^T, \quad \mathbf{r} = [x, y, z]^T \quad (3)$$

where  $\alpha_i$  denotes the position angle, and  $\alpha_1 = -\pi/2$ ,  $\alpha_2 = \pi/2$ ,  $\alpha_3 = 0$ .

Taking  $z$ ,  $\theta$  and  $\psi$  as independent coordinates and considering the constrained conditions arisen by R joints, one can obtain the parasitic motion of the Exe-Variant

$$\begin{cases} x = 0 \\ y = -r_a \sin\varphi \cos\theta \\ \varphi = \arctan(\tan\psi / \sin\theta) \end{cases} \quad (4)$$

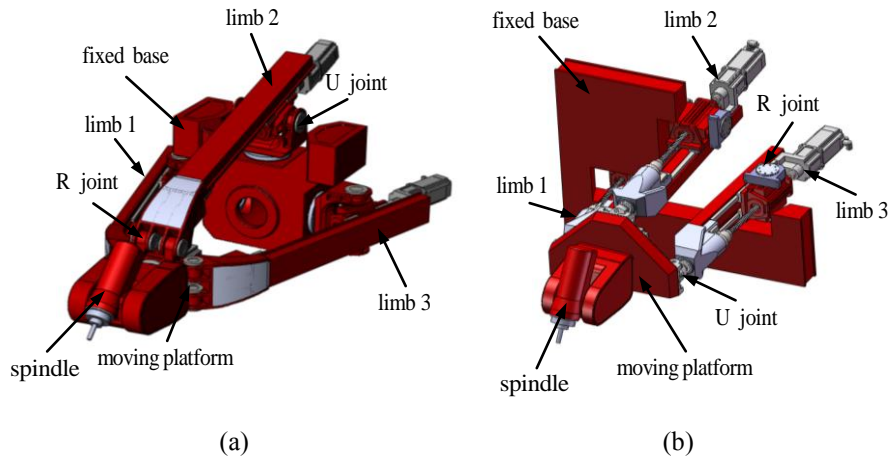
Substituting Eq. (4) into Eq. (2), the inverse kinematic can be solved

$$q_i = \|\mathbf{a}_i - \mathbf{b}_i\|, \quad \mathbf{w}_i = \frac{\mathbf{a}_i - \mathbf{b}_i}{q_i} \quad (5)$$

## 2.2 Conceptual Design

In this subsection, the conceptual design for the Exe-Variant is carried out at the inspiration of that of Exechon machine. As shown in Fig. 2(a), the Exechon machine is a hybrid PKM, which consists of a parallel structure to locate the position of the moving platform and a serial structure to orient the machining tool. The moving platform is supported by three limbs denoted as limb 1, limb 2, and limb 3, respectively. At the home position, limb 1 and limb 2 are symmetrical with

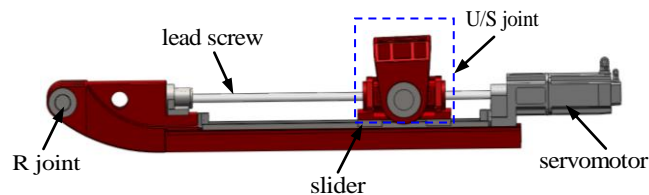
respect to limb 3. Limb 1 (limb 2) connects the base to the moving platform by a universal (U) joint, a prismatic (P) joint and a revolute (R) joint in sequence. Limb 3 is slightly different from limb 1 and limb 2 in that it is allowed to spin by itself.

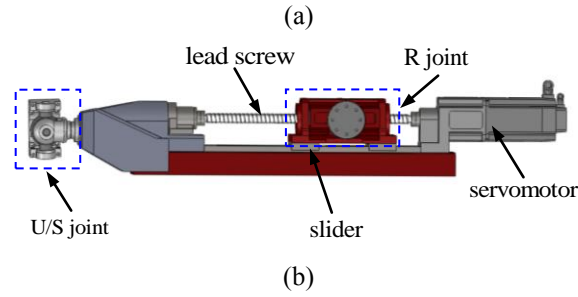


**Fig. 2** Conceptual design of (a) Exechon; (b) Exe-Variant

By exchanging the sequence of kinematic pairs in each limb of the Exechon, a conceptual design of the Exe-Variant can be completed and presented in [Fig. 2\(b\)](#). Herein, Limb 1 (and limb 2) connects the fixed base to the moving platform by a revolute (R) joint, a prismatic (P) joint and a universal (U) joint in sequence. Limb 3 connects the fixed base to the moving platform by a revolute (R) joint, a prismatic (P) joint and a spherical (S) joint in sequence.

In order to improve the reconfigurability and enhance the rigidity, the structure of the limb especially the joints should be delicately designed. For instance, a potential design of the limb structure in Exechon and Exe-Variant are compared as shown in [Fig. 3](#).





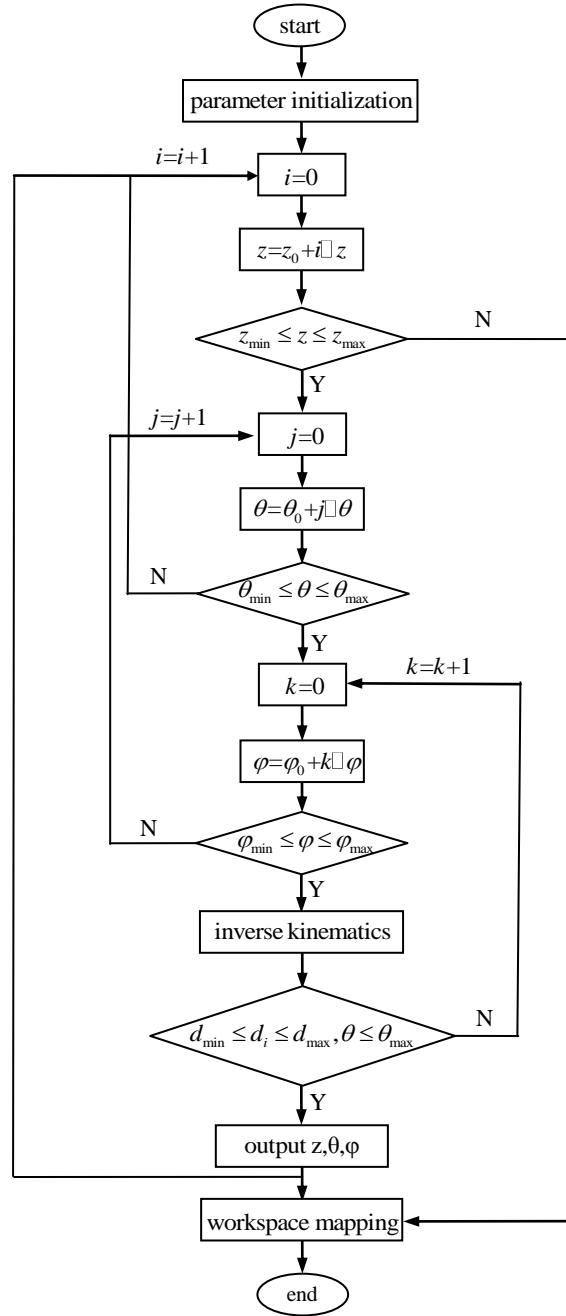
**Fig. 3** Structural design of limb (a) Exechon; (b) Exe-Variant

### 3 Workspace Analyses

Workspace is one of the most important performance indices of parallel mechanisms reflecting their position-orientation capabilities. In this section, an algorithm is proposed to search the reachable workspace for the Exe-Variant which is then compared with that of the Exechon with approximate dimensions.

To facilitate the comparison, the origin of the moving coordinate system is set as the reference point in this paper. The basic idea is ‘slice’ the workspace into discrete pieces of work plane and then ‘partition’ the work plane into grids with different orientation angles. When the three independent coordinates are given, the distance between two joints  $A_i$  and  $B_i$  in an individual limb and rotation angles of limbs can be calculated through the inverse kinematics as demonstrated in Eq. (5). Then the obtained analytical values are compared to the corresponding allowable values of joints. If the analytical values are less than the allowable ones, the given independent coordinates are within the reachable workspace of the moving platform. Otherwise, they are beyond the reachable workspace. The detailed process for the reachable workspace search is shown in the Fig. 4.

For the convenience of comparison, parameters of the Exechon and Exe-Variant example systems are set to be approximate as shown in Table 1. Herein,  $q_{\min}$  and  $q_{\max}$  denote the minimum and maximum distance between the two joints in an individual limb;  $\theta_S$ ,  $\theta_U$  and  $\theta_R$  denote the allowable rotation angles of S joint, U joint and R joint, respectively.



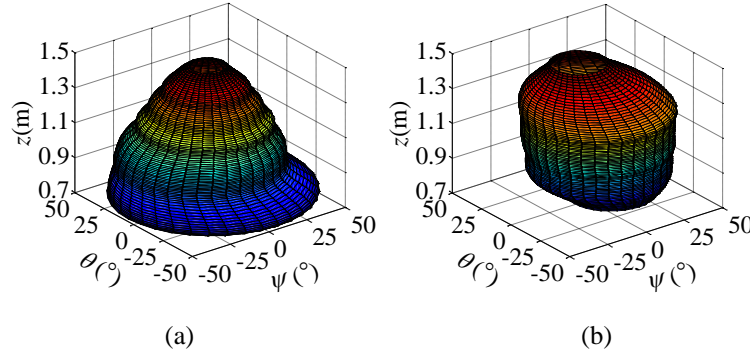
**Fig. 4** Procedure of searching for the reachable workspace



**Table 1** Dimensional parameters of the module.

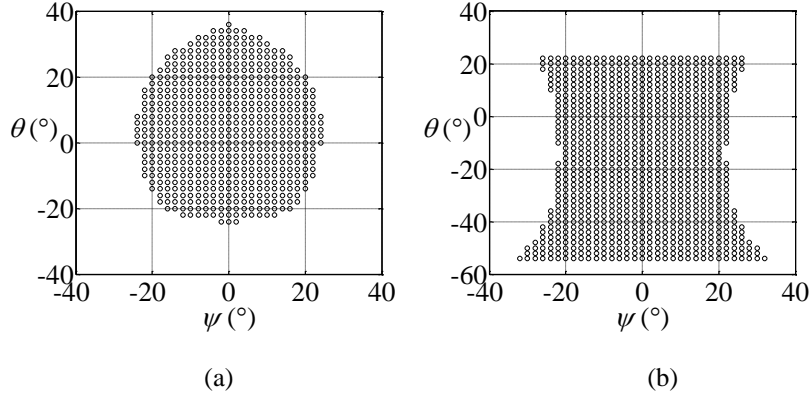
$r_a$	$r_b$	$q_{\min}$	$q_{\max}$	$\theta_S$	$\theta_U$	$\theta_R$
0.34 m	0.45 m	0.8 m	1.5 m	$120^\circ$	$120^\circ$	$120^\circ$

The reachable workspace of the Exechon and the Exe-Variant is depicted in Fig. 5. One can find that the reachable workspace of Exechon decreases monotonously with the increment of  $z$ , while that of Exe-Variant is very different in that it increases firstly with the increment of  $z$ , then almost keeps unchanged and at last decreases with the increment of  $z$ . Further calculation reveals that there are 9659 and 9563 feasible points within the reachable workspace of the Exechon and the Exe-Variant, respectively. This indicates that the Exe-Variant possess a competitive reachable workspace with the Exechon. From the perspective of orientation ability, the Exe-Variant seems to own a better performance than that of the Exechon.

**Fig. 5** Reachable workspace (a) for Exechon; (b) for Exe-Variant

In order to clarify the orientation characteristics of the reachable workspace, the cross-section of the Exechon and the Exe-Variant at the work plane of  $z=1.2$  m are demonstrated and compared in Fig. 6. Clearly, it can be found that the cross-section is symmetric about the line  $\psi = 0^\circ$  for the two PKMs, which is coincident with their structure features that limb 1 and limb 2 are symmetric about

limb 3. As can be seen, the Exe-Variant demonstrates a larger cross-section area at this work plane, indicating a better reachable ability.



**Fig. 6** Cross-section of reachable work plane when  $z=1.2$  m: (a) Exechon; (b) Exe-Variant

## 4 Conclusions

To summarize the contributions of this paper, the following conclusions can be drawn.

- (1) A conceptual design of a new configurable module is proposed at the inspiration of the Exechon machine by exchanging the sequence of kinematic pairs in each limb.
- (2) A kinematic model of the Exe-Variant is established with which the inverse position analysis of the PKM is derived.
- (3) An algorithm is presented to numerically predict the reachable workspace of Exe-Variant and compared with that of the Exechon PKM. The comparison between the two PKMs indicates that the proposed Exe-Variant possesses a competitive workspace with the Exechon.
- (4) Further investigations on the singularity analysis, stiffness formulation and structural enhancement will be carried out in the next step followed by virtual prototype validations.

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